ROBUST CONTROL

S.P.Bhattacharyya
Texas A&M University, Texas, USA

Keywords: control systems, robust control, optimal control, robust performance, robustness, feedback system, unity feedback, stability, robust stability, servomechanism, tracking, disturbance rejection, stabilization, robustification, stability margin, gain margin, phase margin, H_∞ optimal control, l_1 optimal control, absolute stability, integral control, PID controller, state space model, state feedback, observers, closed loop eigenvalues, Internal Model Principle, block diagram, characteristic equation.

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Summary

Robust control is that branch of control theory which deals explicitly with system uncertainty and how it affects the analysis and design of control systems. In this article, we give an overview of robust control. We begin by describing the fundamental property of robustness inherent in a properly designed feedback structure. First, we show how a feedback system can be used to obtain an accurately controlled gain despite large parameter variability. Then, we demonstrate how integral control in a stable feedback system can precisely zero out the steady state error despite large plant uncertainty. This is followed by a short historical account of control theory and robust control. We then trace the central role robustness has played in classical control designs, linear quadratic optimal control methods, H_∞ optimal control techniques, absolute stability methods, and parametric robustness methods. These various approaches develop different analysis and design techniques to address the same basic problem of obtaining precise behavior from physical systems in the presence of significant uncertainty regarding the system models and signals.

1. Introduction and Basic Elements of Control Systems

In this section, we make some introductory and motivational remarks describing the problems of robust stability and control. We begin with some basic control concepts, terminology and techniques. This is followed by two sections dealing with the robustness of feedback systems to large parameter variations. Next, a brief historical sketch of control theory is included to serve as a background for the discussion of robustness. This is followed by a description of various uncertainty models in robust control and some of the current techniques for the design of robust control systems.

To understand the concept of robustness in the context of control systems it is necessary to begin with a description of the basic functioning of a control system. We do so in the brief overview that follows.

Control theory and control engineering deal with dynamic systems such as aircraft, spacecraft, ships, trains and automobiles, chemical and industrial processes such as distillation columns and rolling mills, electrical systems such as motors, generators and power systems, machines such as numerically controlled lathes and robots. In each case, the *setting* of the control problem is:

1. There are certain dependent variables, called *outputs* to be controlled, which must be made to behave in a prescribed way. For instance, it may be necessary to *assign* the temperature and pressure at various points in a process, or the position and velocity of a vehicle, or the voltage and frequency in a power system, to given desired fixed values, despite uncontrolled and unknown variations at other points in the system.
2. Certain independent variables called *inputs*, such as voltage applied to the motor terminals, or valve position, are available to regulate and control the behaviour of the system. Other dependent variables, such as position, velocity or temperature are accessible as dynamic *measurements* on the system.

3. There are unknown and unpredictable *disturbances* impacting the system. These could be, for example, the fluctuations of load in a power system, disturbances such as wind gusts acting on a vehicle, external weather conditions acting on an air conditioning plant or the fluctuating load torque on an elevator motor, as passengers enter and exit.
4. The equations describing the plant dynamics, and the parameters contained in these equations, are not known at all or at best known imprecisely. This uncertainty can arise, even when the physical laws and equations governing a process are known well, for instance, because these equations were obtained by linearizing a nonlinear system about an operating point. As the operating point changes so do the system parameters.

These considerations suggest the general representation of the *plant*, or system to be controlled, shown in Figure 1.

The inputs or outputs shown in Figure 1 could actually be representing a vector of signals. In such cases, the plant is said to be a *multivariable plant*, as opposed to the case, where the signals are scalar, in which case the plant is said to be a *scalar or monovariable plant.*

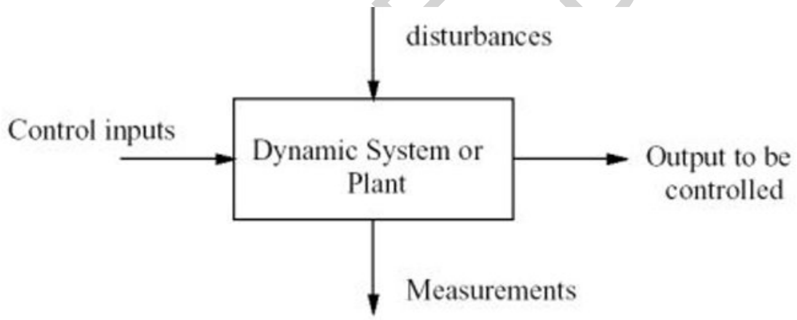

Figure 1: A general plant

Control is exercised by feedback, which means that the corrective control input to the plant is generated by a device which is driven by the available measurements. Thus the controlled system can be represented by the following *feedback* or *closed loop system* shown in Figure 2.

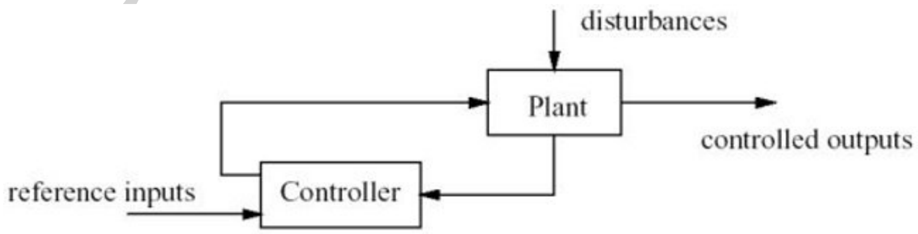

Figure 2: A feedback control system

The control design problem is to determine the characteristics of the controller so that the controlled outputs can be:

1. Made to closely follow or equal prescribed values called references, which may be constant or time-varying,
2. Maintained at the reference values despite the unknown disturbances,
3. Conditions (1) and (2) are met despite the inherent uncertainties and changes in the plant dynamic characteristics.

The first condition is called *tracking*, the second, *disturbance rejection* and the third *robustness* of the system. The simultaneous satisfaction of (1), (2) and (3) is called *robust tracking and disturbance rejection* and control systems designed to achieve this are called *robust servomechanisms*. In the next two sections, we show how the challenging design requirements described here can be met by using properly designed feedback structures.

2. Feedback and Robustness

In this section, we illustrate how robust systems can be built from highly unreliable components by using the feedback structure. Specifically, we consider the problem of obtaining a precisely controlled value of gain from a system containing large parameter uncertainty.

Consider the system shown in Figure 3. Suppose that the system gain G is required to be 100. Due to poor reliability of the components, this can vary by 50%.

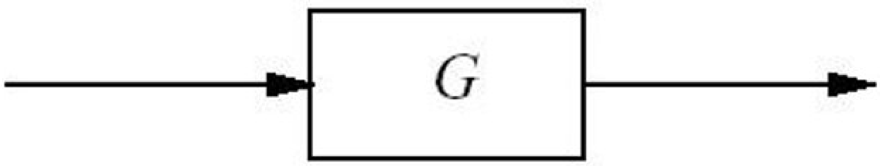

Figure 3: An open loop system

Then the actual gain can range from 50 to 150. To remedy this situation, we use the feedback structure shown in Figure 4.

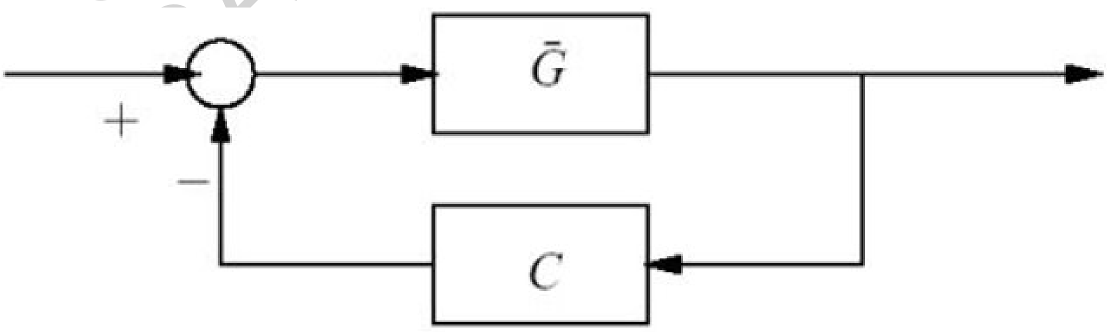

Figure 4: A feedback system

The gain $\bar{G}$ is again made with the same unreliable components but with nominal value much higher than 100, say 10,000, and we set C = 0.01. The overall gain of the feedback system is given by the expression

$$\frac{\bar{G}}{1+C\bar{G}}. \tag{1}$$

It can easily be verified that with 50% variation in $\bar{G}$ (that is, $\bar{G}$ varies from 5,000 to 15,000) the gain of the feedback system varies from 98.039 to 99.338. This remarkable increase in robustness, corresponding to a reduction of uncertainty from 50% to 1%, is one of the main reasons for the widespread use of feedback in the control and electronics industry. In the next section, we illustrate another wonderful application of feedback to the tracking and disturbance rejection problem.

3. Robustness and Integral Control

Integral control is used almost universally in the control industry to design robust servomechanisms. It works magically to remove tracking errors in the presence of disturbances even when very little is known regarding the signals and system characteristics. Integral action is most easily implemented by computer control. It turns out that hydraulic, pneumatic, electronic and mechanical integrators are also commonly used elements in control systems. In this section, we explain how integral control works in general to achieve robust tracking and disturbance rejection.

Let us first consider an integrator, shown in Figure 5. The input-output relationship is given by

Figure 5: An integrator

$$y(t) = K\int_0^t u(\tau)\,d\tau + y(0) \tag{2}$$

or

$$\frac{dy}{dt} = Ku(t) \tag{3}$$

where K is the integrator gain.

Now *suppose* that the output $y(t)$ is a *constant*. It follows from Eq. (3) that

$$\frac{dy}{dt} = 0 = Ku(t)\,\forall t > 0 \qquad (4)$$

Equation (4) proves the following important facts about the operation of an integrator:

1. If the output of an integrator is *constant* over a segment of time, then the input must be identically *zero* over that same segment.
2. The output of an integrator changes as long as the input is nonzero.

The simple fact stated above suggests how an integrator can be used to solve the servomechanism problem. If a plant output $y(t)$ is to track a constant reference value r, despite the presence of unknown constant disturbances, it is enough to:

a. attach an integrator to the plant and make the error

$$e(t) = r - y(t) \qquad (5)$$

the input to the integrator

b. ensure that the closed loop system is asymptotically stable so that under constant reference and disturbance inputs, all signals, including the integrator output, reach constant steady state values.

This is depicted in the block diagram shown in Figure 6:

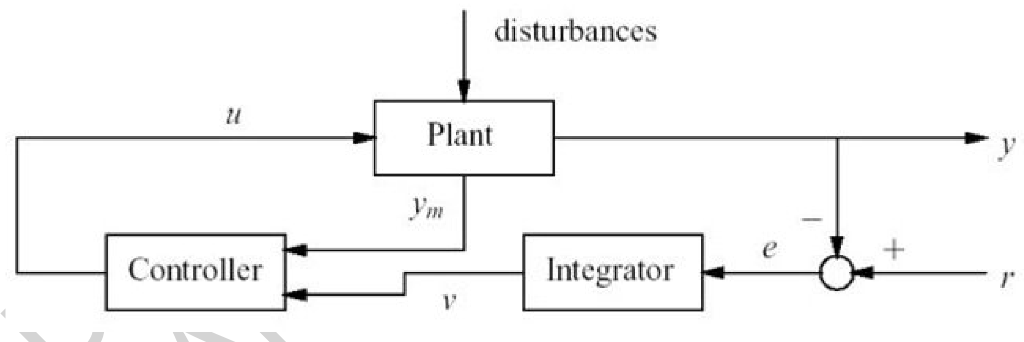

Figure 6: Servomechanism

If the feedback system shown in the block diagram above is asymptotically stable, and the inputs r and d are constant; it follows that all signals in the closed loop will tend toward constant values. In particular the integrator output $v(t)$ tends toward a constant value. Therefore, by the fundamental operation of an integrator established previously, it follows that the integrator input tends toward *zero*. Since we have arranged that this input is the tracking error, it follows that $e(t) = r - y(t)$ tends to zero and hence $y(t)$ tracks r as $t \to \infty$.

We emphasize that the steady state tracking property established previously is *very robust*. It holds as long as the closed loop is asymptotically stable and

(i) is independent of the particular values of the constant disturbances or references,
(ii) is independent of the initial conditions of the plant and controller and
(iii) is independent of whether the plant and controller are linear or nonlinear.

Thus, the tracking problem is reduced to guaranteeing that stability is assured. In many practical systems, the stability of the closed loop system can even be ensured without detailed and exact knowledge of the plant characteristics and parameters, and this is known as *robust stability*.

We next discuss how several plant outputs $y_1(t)$, $y_2(t)$,..., $y_m(t)$ can be pinned down to prescribed but arbitrary constant reference values r_1, r_2, ...,r_m in the presence of unknown but constant disturbances d_1, d_2, $\cdots$, d_q. The previous argument can be extended to this multivariable case by attaching m integrators to the plant, and driving each integrator with its corresponding error input $e_i(t) = r_i - y_i(t)$, $i = 1, \cdots m$. This is shown in the configuration shown in Figure 7:

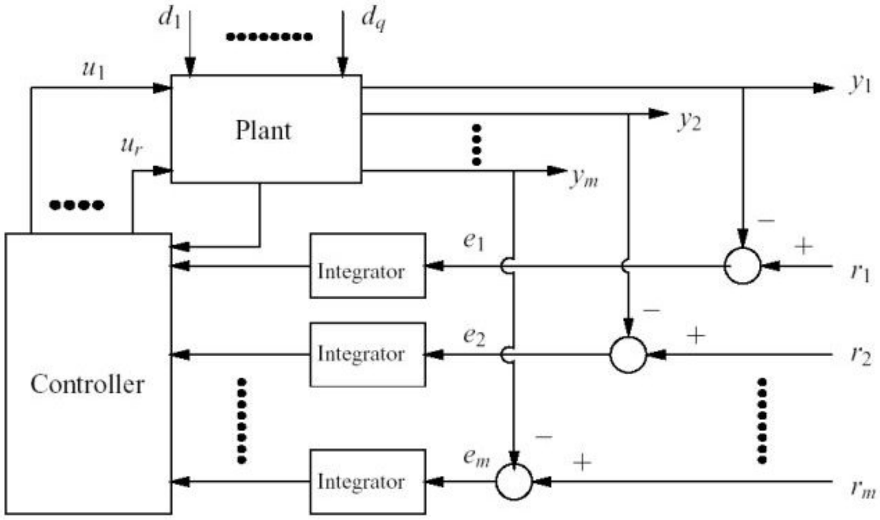

Figure 7: Multivariable servomechanism

Once again it follows that as long as the closed loop system is stable, all signals in the system must tend to constant values and integral action forces the $e_i(t)$, $i = 1, \cdots , m$ to tend to zero asymptotically, regardless of the actual values of the disturbances d_j, $j = 1, \cdots , q$. The existence of steady state inputs u_1, u_2, ...$\cdot$, u_r that make $y_i = r_i$, $i = 1,..., m$ for arbitrary r_i , $i = 1, \cdots , m$ requires that the plant equations relating y_i, $i = 1, \cdots , m$ to u , $j = 1, \cdots , r$ be invertible for constant inputs. In the case of linear time invariant systems, this is equivalent to the requirement that the corresponding transfer matrix have rank equal to m at $s = 0$. Sometimes, this is restated as the two conditions

(i) $r \geq m$ or at least as many control inputs as outputs to be controlled and
(ii) $G(s)$ has no transmission zero at $s = 0$.

In general, the addition of an integrator to the plant tends to make the system less stable. This is because the integrator is an inherently unstable device; for instance, its response to a step input, a bounded signal, is a ramp, an unbounded signal. Therefore, the problem of stabilizing the closed loop becomes a critical issue even when the plant is initially stable.

Since integral action and the attainment of zero steady state error is *independent* of the particular value of the integrator gain K, we can see that this gain can be used to try to stabilize the system. This single degree of freedom is sometimes insufficient for attaining stability and an acceptable transient response, and additional gains are introduced as explained in the next section. This leads naturally to the proportional integral derivative (PID) controller structure of Figure 8 commonly used in industry:

As long as the closed loop is stable, it is clear that the input to the integrator will be driven to zero independent of the values of the gains. Thus, the function of the gains k_p, k_i and k_d is to stabilize the closed loop system, if possible, and to adjust the transient response of the system. In general, the derivative can be computed or obtained if the error is varying slowly.

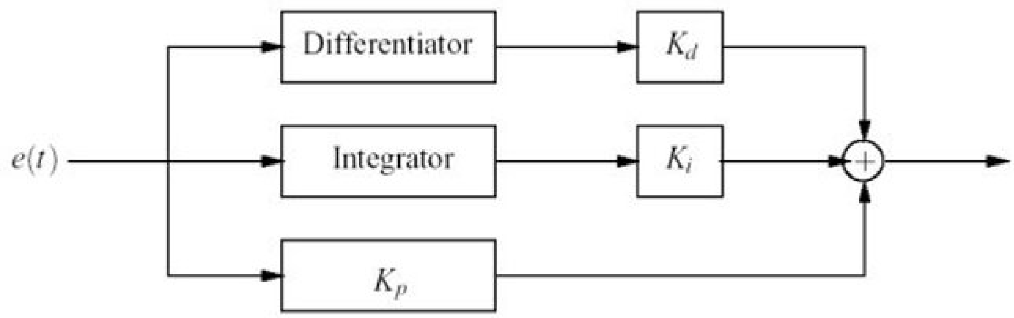

Figure 8: PID controller

Since the response of the derivative to high frequency inputs is much higher than its response to slowly varying signals, the derivative term is usually omitted, especially if the error signal is corrupted by high frequency noise.

In the next section, we provide a short history of control theory and robust control. This will serve as a useful backdrop to the discussion of robustness that will follow.

4. A Short History of Control Theory and Robust Control

4.1. The Classical Period

Control theory began with the publication of Maxwell's paper "On Governors" in 1868. This paper was motivated by the need to understand and correct the observed unstable behaviors of many locomotive steam engines in operation at the time. Maxwell showed that the behavior of a dynamic system could be approximated in the vicinity of an equilibrium point by a linear differential equation. Consequently, the stability or instability of such a system could be determined from the location of the roots of the *characteristic equation* of this linear differential equation. The speed of locomotives was controlled by centrifugal governors and so the problem was to determine the design parameters of the controller (flyball mass and inertia, spring tension etc.) to ensure stability of the closed loop system. Maxwell posed this in general terms: Determine the constraints on the coefficients of a polynomial that ensure that the roots are confined to the left half plane, the stability region for continuous time systems.

This problem had actually been already solved for the first time by the French mathematician Hermite in 1856! In his proof, Hermite related the location of the zeros of a polynomial with respect to the real line to the signature of a particular quadratic form. In 1877, the English physicist E.J. Routh, using the theory of Cauchy indices and of Sturm sequences, gave his now famous algorithm to compute the number k of roots which lie in the right half of the complex plane $\mathrm{Re}(s) \geq 0$. This algorithm thus gave a necessary and sufficient condition for stability in the particular case when $k = 0$. In 1895, A. Hurwitz drawing his inspiration from Hermite's work, gave another criterion for the stability of a polynomial. This new set of necessary and sufficient conditions took the form of n determinantal inequalities, where n is the degree of the polynomial to be tested. Equivalent results were discovered at the beginning of the century by I. Schur and A. Cohn for the discrete-time case, where the stability region is the interior of the unit disc in the complex plane.

One of the main concerns of control engineers had always been the analysis and design of systems that are subjected to various types of uncertainties or perturbations. These may take the form of noise or of some particular external disturbance signals. Perturbations may also arise within the system, in its physical parameters. This latter type of perturbations, termed parametric perturbations, may be the result of actual variations in the physical parameters of the system, due to aging or changes in the operating conditions. For example in aircraft design, the coefficients of the models that are used depend heavily on the flight altitude. It may also be the consequence of uncertainties or errors in the model itself; for example, the mass of an aircraft varies between an upper limit and a lower limit depending on the passenger and baggage loading. From a design standpoint, this type of parameter variation problem is also encountered when the *controller structure* is fixed, but its parameters are adjustable. The choice of controller structure is usually dictated by physical, engineering, hardware and other constraints such as simplicity and economics. In this situation, the designer is left with a restricted number of controller or design parameters that have to be adjusted so as to obtain a satisfactory behavior for the closed-loop system; for example, PID controllers have only three parameters that can be adjusted.

The characteristic polynomial of a closed-loop control system containing a plant with uncertain parameters will depend on those parameters. In this context, it is necessary to analyze the stability of a *family* of characteristic polynomials. It turns out that the Routh-Hurwitz conditions, which are so easy to check for a single polynomial, are almost useless for families of polynomials because they lead to conditions that are highly nonlinear in the unknown parameters. Thus, in spite of the fundamental need for dealing with systems affected by parametric perturbations, engineers were faced from the outset with a major stumbling block in the form of the nonlinear character of the Routh-Hurwitz conditions, which moreover was the only tool available to deal with this problem.

One of the most important and earliest contributions to stability analysis under parameter perturbations was made by Nyquist in his classic paper of 1932 on feedback amplifier stability. This problem arose directly from his work on the problems of long-distance telephony. This was soon followed by the work of Bode which eventually led to the introduction of the notions of *gain* and *phase margins* for feedback systems. Nyquist's criterion and the concepts of gain and phase margin form the basis for much of the classical control system design methodology and are widely used by practicing control engineers.

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Biographical Sketch

Shankar P. Bhattacharyya was born in Rangoon, Burma, on June 23, 1946. He obtained his undergraduate education (1962-1967) at the Indian Institute of Technology, Bombay, and graduate education, (M.S., Ph.D, 1967-1971) at Rice University, Houston. He established the graduate Program in Automatic Control at the Federal University, Rio de Janeiro, Brazil during 1971-1980 and was Chairman of the Department of Electrical Engineering from 1978 to 1980. In 1974-1975 he was a National Academy of Sciences Research Fellow at the NASA Marshall Space Flight Center. In 1980 he joined Texas A&M University, College Station, where he is presently Professor of Electrical. Engineering From 1990-1992 he served as Director of the Systems and Control Institute. In 1990, 1991 and 1992, respectively, he received the Texas Engineering Experiment Station Fellowship, the Halliburton Award of Excellence and the Dresser Industries Professorship from Texas A&M University. He served as an Associate Editor of the IEEE Transactions on Automatic Control in 1985-1986, and a member of the IEEE Control Systems Society Board of Governors. He was a Senior Fullbright Lecturer in 1989 and a UNDP consultant to the Government of India in 1990. In 1998 he received the Boeing Welliver Faculty Award. He held visiting lecturing assignments in Italy, Germany, France, India, Brazil, Mexico and Japan. In 1989 he was elected a Fellow of the IEEE for contributions to linear control system analysis and design. He has made a number of original contributions to Control Theory and Engineering. These are documented in 4 books, 80 journals and over 100 conference papers in the field of Automatic Control, authored or co-authored by him. Dr. Bhattacharyya other interests include playing Indian classical music on the Sarode. He is a disciple, since 1982, of the renowned Sarode maestro Ustad Ali Akbar Khan and regularly performs as a concert artist in USA, India and Europe.